62[2.10].—T. N. L. PATTERSON, "Gaussian Formula for the Calculation of Repeated Integrals," tables appearing in the microfiche section of this issue.

Abscissas and weights of the r-point Gaussian quadrature formula for the integral

$$(n-1)!\int_{-1}^{1}dx_1\int_{0}^{x_1}dx_2\int_{0}^{x_2}dx_3\cdots\int_{0}^{x_{n-1}}f(x_n)dx_n \equiv \int_{-1}^{1}w(x)f(x)dx$$

are tabulated to 20 significant figures for $n = 2, 4$; $r = 2(2)16$ and $n = 3, 5$; $r = 2(1)16$. The resulting formula is exact if $f(x)$ is a polynomial of degree $2r - 1$ (or $2r$ when r is even). The weighting function is

$$w(x) = (-1)^n w(-x) = (1-x)^{n-1}, \qquad 0 < x \leqq 1\,.$$

This has discontinuous even (odd) derivatives at $x = 0$.

A normal approach to such an integration might be to divide the interval into two sections and use either the Gauss-Legendre formula or better still the appropriate Gauss-Jacobi formula in each section separately. However, the existence of these tables does allow the interval $[-1, 1]$ to be treated as a whole.

J. N. L.

63[2.20, 7].—HENRY E. FETTIS & JAMES C. CASLIN, *More Zeros of Bessel Function Cross Products*, Report ARL 68-0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1968, v + 56 pp., 28 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.]

In this compact report the authors continue their previous 10D tabulation [1] of the roots of the equations (a) $J_0(\alpha)Y_0(k\alpha) = Y_0(\alpha)J_0(k\alpha)$, (b) $J_1(\alpha)Y_1(k\alpha) = Y_1(\alpha)J_1(k\alpha)$, and (c) $J_0(\alpha)Y_1(k\alpha) = Y_0(\alpha)J_1(k\alpha)$.

These new tables give the roots α_n and the corresponding normalized roots γ_n of all three equations, for $n = 5(1)10$ and $k = 0.001(0.001)0.3$. For equation (c) these roots are also tabulated corresponding to $k^{-1} = 0.001(0.001)0.3$.

The normalized roots are related to the others by the equation $\gamma_n = (1 - k)\alpha_n/(n\pi)$ for (a) and (b), and by $\gamma_n = |k - 1|\alpha_n/[(n - \frac{1}{2})\pi]$ for (c). The authors note the properties $\lim_{k\to 1}\gamma_n = 1$ (all n) and $\lim_{n\to\infty}\gamma_n = 1$ (all k).

For examples of applications of these tables, as well as details of their calculation, the user should consult the earlier report [1].

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66-0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966. (See *Math. Comp.*, v. 21, 1967, pp. 507–508, RMT **64**.)

64[2.40, 7, 10].—JOHN RIORDAN, *Combinatorial Identities*, John Wiley & Sons, Inc., New York, 1968, xii + 256 pp., 23 cm. Price $15.00.

This volume deals in the main with identities involving the binomial coefficients. As is well known, binomial coefficients are the simplest combinatorial entities and arise quite naturally in a wide variety of combinatorial problems.

technique discussed by Patterson [1] When n is even it can be shown that only even point formulae exist Tables 1 to 4 in the microfiche section give the results The r point formulae have been checked by verifying that they integrate exactly all powers up to degree 2r-1 (2r when r is even) and are correct to all digits given. In the tables the ± indication before the weights indicates that the weights and abscissae have the same sign (n even) In the absence of this indication all the weights are to be taken positive (n odd). In addition, the factor $1/(n-1)!$ appearing in (1) has not been included in the values given for the weights so that the value of the repeated integral is obtained after dividing the results obtained by application of the formulae by $(n-1)!$ A few low degree formulae of this type have earlier been given by Kopal, Carrus, and Kavanagh [2]

References

1 T N L. Patterson, "The optimum addition of points to quadrature formulae", Maths. Comp., 22, 847, 1968

2 Z Kopal, P. Carrus, and K E. Kavanagh, J. Maths. Phys 30, 44, 1951

Acknowledgement: This research has been supported by the Office of Naval Research under Contract N00014-67-A-0310-004.

TABLE 1.1 A=0, 2 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.54772 25575 05166 11346(0)	.30429 03097 25092 28525(0)

TABLE 1.2 A=0, 4 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.79200 59217 60865 11701(0)	.69534 28804 71541 79905(-1)
.37214 56511 38755 30639(0)	.29986 94151 77678 22091(0)

TABLE 1.3 A=0, 6 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.88253 10979 63248 54938(0)	.22651 33587 24697 94301(-1)
.62686 01608 87658 43172(0)	.11585 16504 33743 00238(0)
.28209 00214 17742 07481(0)	.26251 68155 42157 58373(0)

TABLE 1.4 A=0, 8 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.92488 39673 29570 09135(0)	.93443 20777 56612 39800(-2)
.75636 81074 02448 31184(0)	.51020 49667 63126 24650(-1)
.51408 06061 43694 53998(0)	.13136 90234 71769 09638(0)
.22732 88824 80793 90109(0)	.22830 23164 62705 51116(0)

TABLE 1.5 A=0,10 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.94792 86287 01912 73213(0)	.45107 92165 49736 54183(-2)
.82929 80447 85324 20380(0)	.25451 96697 40678 16041(-1)
.65355 58761 15119 82249(0)	.69321 74041 57130 64115(-1)
.43440 49427 27350 20981(0)	.13301 63257 25256 62329(0)
.19048 41804 26529 07300(0)	.20051 48719 02264 98045(0)

TABLE 1.6 A=0,12 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.96181 29567 55326 91264(0)	.24324 03036 88232 04667(-2)
.87403 36979 41467 07344(0)	.13987 40458 11421 71088(-1)
.74178 66536 88679 69348(0)	.39305 80047 94310 48604(-1)
.57243 66400 98118 09625(0)	.79042 65622 44957 16203(-1)
.37565 28865 18174 10477(0)	.12904 09387 94109 26658(0)
.16398 11690 99421 74179(0)	.17821 34815 89277 33318(0)

TABLE 1.7 A=0,14 POINT 2 DIMENSIONAL FORMULA.

±ABSCISSAE	±WEIGHTS
.97081 12198 58451 34636(0)	.14235 18593 43366 79138(-2)
.90333 61608 22634 61173(0)	.82842 93283 66326 54358(-2)
.80060 64071 10960 83601(0)	.23735 24912 76210 28311(-1)
.66693 27097 09578 68973(0)	.49082 11807 40882 39734(-1)
.50798 16905 46296 69306(0)	.83339 18197 51532 59855(-1)
.33070 98113 35014 57706(0)	.12298 02576 13653 86720(0)
.14399 12886 46431 11073(0)	.16013 87035 14897 52620(0)

TABLE 1.8 A=0,16 POINT 2 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.97697 05854 41102 99061(0)	.88712 99424 88198 83711(-3)
.92353 14840 18396 96180(0)	.52043 39585 47596 11173(-2)
.84159 94425 55239 90990(0)	.15104 55442 40144 11815(-1)
.73386 22714 90145 90259(0)	.31810 02991 27872 54493(-1)
.60386 99343 41739 73811(0)	.55353 39437 86042 84820(-1)
.45596 61732 00108 72677(0)	.84445 35253 00123 23443(-1)
.29528 52684 65271 59773(0)	.11635 42539 47971 74366(0)
.12837 03842 68668 40156(0)	.14527 69506 05985 55003(0)

TABLE 2.1 A=0, 2 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.31622 77660 16837 93320(0)	.33333 33333 33333 33333(0)

TABLE 2.2 A=0, 3 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.53452 24838 24848 76937(0)	.11666 66666 66666 66667(0)
.00000 00000 00000 00000(0)	.43333 33333 33333 33333(0)

TABLE 2.3 A=0, 4 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.66449 45298 23700 79634(0)	.45777 43316 26915 63485(-1)
.21360 36212 56443 10434(0)	.28755 59001 70641 76985(0)

TABLE 2.4 A=0, 5 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.74897 37382 67244 41712(0)	.19644 28650 36439 34708(-1)
.38701 69777 14003 93413(0)	.14897 39804 75278 78192(0)
.00000 00000 00000 00000(0)	.32943 01327 08821 23341(0)

TABLE 2.5 A=0, 6 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.80506 93913 12211 66938(0)	.93371 56329 69703 27402(-2)
.51184 94949 80571 60109(0)	.79036 70802 05580 06065(-1)
.16382 97723 39671 43680(0)	.24495 94689 83078 29453(0)

TABLE 2.6 A=0, 7 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.84470 59860 59491 33038(0)	.47664 39561 77411 85131(-2)
.60484 62396 64802 77649(0)	.43165 84008 09006 05136(-1)
.30551 45857 47115 41184(0)	.15149 69422 10715 76756(0)
.00000 00000 00000 00000(0)	.26780 82229 59885 68425(0)

TABLE 2.7 A=0, 8 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.87330 68456 92833 15651(0)	.26055 20302 11962 17439(-2)
.67398 90075 18818 40141(0)	.24726 64303 02042 43293(-1)
.41693 05407 03967 72770(0)	.93928 97418 92809 74534(-1)
.13361 40770 13472 93502(0)	.21207 21958 11728 49376(0)

TABLE 2.8 A=0, 9 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.89481 21171 88552 67297(0)	.14982 90927 68029 06847(-2)
.72716 71207 33693 84280(0)	.14693 33172 31540 49867(-1)
.50614 67537 37683 90992(0)	.58799 82699 61559 95971(-1)
.25312 97009 24663 24975(0)	.14515 31955 79666 81939(0)
.00000 00000 00000 00000(0)	.22637 73762 13352 35484(0)

TABLE 2.9 A=0,10 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.91123 58854 64930 14695(0)	.90356 89997 04292 86671(-3)
.76838 49647 09292 55736(0)	.90814 51602 96603 14023(-2)
.58697 63210 36056 50619(0)	.37761 06816 21721 31179(-1)
.35203 92516 58370 50612(0)	.98947 92483 21516 12907(-1)
.11310 72568 64736 79348(0)	.18663 93197 36339 26498(0)

TABLE 2.10 A=0,11 POINT 3 DIMENSIONAL FORMULA.

±ABSCISSAE	WEIGHTS
.92414 62448 06993 09885(0)	.56539 83454 65893 19669(-3)
.80117 36709 37245 81185(0)	.57887 44118 57074 54691(-2)
.63441 60997 51996 35912(0)	.24758 42788 43015 38358(-1)
.43483 60156 21309 24284(0)	.67586 98072 57452 94151(-1)
.21642 31006 78277 20827(0)	.13643 36603 39825 57195(0)
.00000 00000 00000 00000(0)	.19640 02438 38848 58042(0)

TABLE 2.11 A=0,12 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.93441 24389 09135 79701(0)	.36627 90753 26912 22118(-3)
.82746 81084 18986 50693(0)	.38050 29952 78652 50044(-2)
.68108 57911 42138 35847(0)	.16636 08536 68327 23891(-1)
.50342 55107 07277 01608(0)	.46858 86578 33625 17014(-1)
.30481 69927 44023 80929(0)	.99115 64431 76463 18001(-1)
.98201 46327 17701 70435(-1)	.16655 14288 37378 33720(0)

TABLE 2.12 A=0,13 POINT 3 DIMENSIONAL FORMULA.

±ABSCISSAE	WEIGHTS
.94275 16063 16312 73684(0)	.24398 83282 68455 19849(-3)
.84897 85901 64045 81370(0)	.25638 78894 46341 90272(-2)
.71968 56842 42966 62700(0)	.11403 71999 88425 92444(-1)
.56108 41307 19324 67427(0)	.32896 10290 29358 60840(-1)
.38105 80997 78062 17100(0)	.71910 87978 61116 31452(-1)
.18919 36570 30449 17525(0)	.12750 54577 81894 13600(0)
.00000 00000 00000 00000(0)	.17361 86112 81634 47674(0)

TABLE 2.13 A=0,14 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.94958 50305 49491 89737(0)	.16686 03496 16039 65801(-3)
.86669 85675 80376 51977(0)	.17698 40779 84665 98009(-2)
.75173 94338 84506 30066(0)	.79821 92143 10182 94806(-2)
.60950 94010 74146 94888(0)	.23471 48843 66620 52878(-1)
.44611 87708 02813 16463(0)	.52655 59171 86208 58259(-1)
.26888 27298 20592 79873(0)	.96952 16373 46417 27140(-1)
.86845 04852 00877 93291(-1)	.15033 51961 70844 16612(0)

TABLE 2.14 A=0,15 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.95527 67047 63769 36533(0)	.11661 71376 83291 93705(-3)
.88152 52014 54979 15441(0)	.12463 70912 92980 67694(-2)
.77874 53413 31479 53184(0)	.56849 74916 77607 99652(-2)
.65070 98675 19913 68143(0)	.16975 02755 28040 98295(-1)
.50224 20197 88773 59155(0)	.38859 20297 68946 18364(-1)
.33910 67907 39301 10516(0)	.73511 83701 91669 76201(-1)
.16815 40519 23207 86474(0)	.11909 96429 47505 90142(0)
.00000 00000 00000 00000(0)	.15567 93197 39145 12077(0)

TABLE 2.15 A=0,16 POINT 3 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.96004 99057 84450 46212(0)	.83199 45199 93351 26929(-4)
.89400 32842 27444 01706(0)	.89487 44579 50992 74685(-3)
.80159 39596 50264 15825(0)	.41201 74324 74402 05263(-2)
.68582 31067 54780 98092(0)	.12460 00303 52087 41651(-1)
.55054 29051 26961 41543(0)	.29001 04080 27823 69169(-1)
.40035 34653 53368 40629(0)	.56068 79570 99432 16240(-1)
.24060 41327 26881 95640(0)	.93717 04643 10982 90035(-1)
.77889 17689 21388 35550(-1)	.13698 81991 19606 36784(0)

TABLE 3.1 A=0, 2 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.37796 44730 09227 22721(0)	.13228 75655 53229 52953(0)

TABLE 3.2 A=0, 4 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.64725 27891 71299 40203(0)	.15591 94489 91786 67674(-1)
.27026 36426 44810 61851(0)	.14766 34806 84488 12064(0)

TABLE 3.3 A=0, 6 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.77671 81947 40316 73139(0)	.26074 65900 64534 88677(-2)
.51197 00042 40048 33414(0)	.33615 13278 25128 13407(-1)
.21408 91124 43832 72244(0)	.14370 08812 28522 18842(0)

TABLE 3.4 A=0, 8 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.84669 79192 10263 58816(0)	.59026 02501 32896 70044(-3)
.65659 03489 16442 91556(0)	.86589 54803 99283 39041(-2)
.42575 04257 12103 48343(0)	.46188 83830 31136 49670(-1)
.17844 64893 49148 90918(0)	.13533 42631 49692 02946(0)

TABLE 3.5 A=0,10 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.88844 36409 17643 62650(0)	.16717 87614 15869 58817(-3)
.74675 86752 02572 18846(0)	.26263 78730 02782 90225(-2)
.56837 99176 62797 25793(0)	.15522 06235 83472 55655(-1)
.36523 90914 96025 32888(0)	.53876 09244 21387 66072(-1)
.15348 59762 11871 72822(0)	.12633 15182 79069 69660(0)

TABLE 3.6 A=0,12 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.91525 49435 19737 75625(0)	.56012 75350 91124 18695(-4)
.80606 99835 75240 29574(0)	.91678 96965 69947 82594(-3)
.66572 40176 44377 84134(0)	.57470 41222 84461 75805(-2)
.50090 98695 01918 73034(0)	.21674 10080 81643 34791(-1)
.32020 86317 41521 16828(0)	.58200 75497 92926 28571(-1)
.13490 53545 66139 74583(0)	.11779 10168 78666 99186(0)

TABLE 3.7 A=0,14 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.93346 80697 57611 42051(0)	.21362 75359 66930 12979(-4)
.84694 98021 92974 77112(0)	.35906 42506 90061 54904(-3)
.73426 59997 37024 25578(0)	.23367 22399 84845 62880(-2)
.59945 54366 70108 82238(0)	.92682 43056 19953 63441(-2)
.44771 27766 68985 92712(0)	.26654 12144 68127 87699(-1)
.28530 08062 18323 08616(0)	.60346 28763 02617 77982(-1)
.12047 93156 94880 78634(0)	.11001 09122 11955 67891(0)

TABLE 3.8 A=0,16 POINT 4 DIMENSIONAL FORMULA.

± ABSCISSAE	± WEIGHTS
.94639 36583 05592 99615(0)	.90276 45311 74781 13321(-5)
.87623 90439 06789 58748(0)	.15452 36075 41574 89977(-3)
.78405 00814 68042 69256(0)	.10314 78521 55046 12217(-2)
.67239 60637 34534 08553(0)	.42305 83407 74290 52929(-2)
.54462 68651 07984 69752(0)	.12705 36103 82442 30455(-1)
.40472 93358 73397 06318(0)	.30473 28272 20438 66094(-1)
.25740 22556 26185 74918(0)	.61101 30860 31999 83982(-1)
.10892 60972 64733 15633(0)	.10302 37144 39385 75693(0)

TABLE 4.1 A=0, 2 POINT 5 DIMENSIONAL FORMULA.

ABSCISSAE	WEIGHTS
.21821 78902 35992 38127(0)	.20000 00000 00000 00000(0)

TABLE 4.2 A=0, 3 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.40824 82904 63863 01637(0)	.57142 85714 28571 42857(-1)
.00000 00000 00000 00000(0)	.28571 42857 14285 71429(0)

TABLE 4.3 A=0, 4 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.53929 60250 08225 75618(0)	.17489 83763 43154 14807(-1)
.15592 08535 18194 58052(0)	.18251 01623 65684 58519(0)

TABLE 4.4 A=0, 5 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.63460 43088 22788 64312(0)	.56952 95793 36857 82678(-2)
.30295 68710 70859 20296(0)	.78774 88867 68158 91917(-1)
.00000 00000 00000 00000(0)	.23105 96310 59631 05963(0)

TABLE 4.5 A=0, 6 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.70309 10871 17172 21127(0)	.20661 01100 23781 85562(-2)
.41858 40144 15349 51328(0)	.33965 16310 03359 52739(-1)
.12473 92855 22356 26866(0)	.16396 87357 99426 22870(0)

TABLE 4.6 A=0, 7 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.75475 92206 38644 60775(0)	.80689 79372 94357 90738(-3)
.51144 99866 11100 16815(0)	.14799 71961 88602 23603(-1)
.24542 66733 65184 85810(0)	.86210 48498 94569 49046(-1)
.00000 00000 00000 00000(0)	.19636 57949 08776 93889(0)

TABLE 4.7 A=0, 8 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.79391 06521 34516 62595(0)	.34201 28098 37081 11465(-3)
.58429 95806 18647 02553(0)	.67847 18247 66531 51179(-2)
.34632 68998 59823 19683(0)	.44667 42318 85540 41861(-1)
.10501 32836 77070 57042(0)	.14820 58457 53943 56191(0)

TABLE 4.8 A=0, 9 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.82464 42187 87094 93553(0)	.15378 92637 86454 08098(-3)
.64308 28482 60939 31431(0)	.32270 97228 46310 31047(-2)
.43171 30858 26740 43703(0)	.23048 88161 22936 95404(-1)
.20781 39362 69673 16360(0)	.87732 71056 83616 31826(-1)
.00000 00000 00000 00000(0)	.17167 50426 54190 23117(0)

TABLE 4.9 A=0,10 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.84890 87113 92501 80931(0)	.73494 27825 87921 58726(-4)
.69032 24997 72550 87677(0)	.16100 15058 83350 48273(-2)
.50220 60360 86560 91903(0)	.12224 67040 30611 42839(-1)
.29689 24983 66554 60212(0)	.51043 12396 61975 35268(-1)
.91116 43933 82234 42694(-1)	.13504 86962 93649 02491(0)

TABLE 4.10 A=0,11 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.86857 42407 01436 73808(0)	.36775 45200 57685 39654(-4)
.72918 29926 83296 69485(0)	.83257 70785 36237 83477(-3)
.56152 09452 72051 96987(0)	.66162 29545 85606 30524(-2)
.37474 79564 35045 50366(0)	.29433 91269 88563 91254(-1)
.18088 52604 38449 56375(0)	.86604 69452 08608 00821(-1)
.00000 00000 00000 00000(0)	.15295 16214 07769 47700(0)

TABLE 4.11 A=0,12 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.88459 25674 02061 74226(0)	.19275 39255 46379 25121(-4)
.76116 68720 97905 98389(0)	.44801 85685 26297 06285(-3)
.61107 04832 41212 84308(0)	.36911 76290 51445 27785(-2)
.44121 12880 35784 08547(0)	.17248 77817 40816 33027(-1)
.26052 90291 65839 24785(0)	.54597 45972 10820 56614(-1)
.80686 41558 15391 32856(-1)	.12399 52918 53240 92259(0)

TABLE 4.12 A=0,13 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.89790 49602 13747 43130(0)	.10476 11653 96328 28041(-4)
.78798 89750 44683 59968(0)	.24866 05892 14883 06549(-3)
.65316 57495 21509 35685(0)	.21077 62922 27654 06148(-2)
.49875 50138 30386 85489(0)	.10226 83764 91089 95900(-1)
.33167 95251 82639 75272(0)	.34063 35328 39805 17825(-1)
.16048 90298 65786 61081(0)	.84265 00696 55020 51126(-1)
.00000 00000 00000 00000(0)	.13815 58049 46754 75808(0)

TABLE 4.13 A=0,14 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.90901 39893 99753 68780(0)	.59041 44100 36267 44635(-5)
.81052 08160 39841 50663(0)	.14258 55437 58539 37337(-3)
.68885 51044 97629 24892(0)	.12371 43411 20658 61523(-2)
.54816 04073 93710 91373(0)	.61889 00755 70970 16444(-2)
.39386 97195 17881 87819(0)	.21464 61003 53986 40625(-1)
.23248 85005 22008 78518(0)	.56353 42672 28643 91276(-1)
.72520 06346 03962 68478(-1)	.11460 74293 86961 77825(0)

TABLE 4.14 A=0,15 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.91843 27656 71018 28888(0)	.34258 93455 08568 52570(-5)
.82973 76648 75286 34956(0)	.83920 49900 87560 44505(-4)
.71954 54229 67212 44130(0)	.74212 68505 46761 18679(-3)
.59112 95669 86247 09425(0)	.38049 59736 08751 66315(-2)
.44883 73087 90765 86718(0)	.13619 04030 42521 04272(-1)
.29783 51484 69431 28931(0)	.37288 82547 45310 02700(-1)
.14442 84991 05030 83885(0)	.81399 99250 21219 62582(-1)
.00000 00000 00000 00000(0)	.12611 54174 79993 62180(0)

TABLE 4.15 A=0,16 POINT 5 DIMENSIONAL FORMULA.

± ABSCISSAE	WEIGHTS
.92644 50539 91680 79416(0)	.20465 53042 26139 65911(-5)
.84615 88184 05082 65440(0)	.50737 01383 12193 79274(-4)
.74593 26595 35321 34628(0)	.45591 67037 90826 56992(-3)
.62837 62046 81434 68309(0)	.23861 26595 69517 76827(-2)
.49700 16064 46146 63183(0)	.87666 25944 91162 11118(-2)
.35596 22029 22517 50073(0)	.24830 50032 97504 68465(-1)
.21012 58816 79671 72266(0)	.56962 90135 10565 73172(-1)
.65927 58950 41801 47343(-1)	.10654 51455 07921 85222(0)

THE OPTIMUM ADDITION OF POINTS TO QUADRATURE FORMULAE

BY

T. N. L. PATTERSON

This amended table is to be substituted for table M-14 in the microfiche supplement by same author Vol.22, no.104

TABLE M14. GENERAL EXTENDED FORMULA OF DEGREE 191. (AMENDED)

ABSCISSAE	WEIGHTS
.99998 24303 54891 59858(0)	.50536 09520 78625 17625(-4)
.99987 28881 20357 61194(0)	.18073 95644 45388 35782(-3)
.99959 87996 71910 68325(0)	.37774 66463 26984 66027(-3)
.99909 81249 67667 59766(0)	.63260 73193 62633 54422(-3)
.99831 66353 18407 39253(0)	.93836 98485 42381 50079(-3)
.99720 62593 72221 95908(0)	.12895 24082 61041 73921(-2)
.99572 41046 98407 18851(0)	.16811 42865 42146 99063(-2)
.99383 19632 12755 02221(0)	.21088 15245 72663 28793(-2)
.99149 57211 78106 13240(0)	.25687 64943 79402 03731(-2)
.98868 47575 47429 47994(0)	.30577 53410 17553 11361(-2)
.98537 14995 98520 37111(0)	.35728 92783 51729 96494(-2)
.98153 11495 53740 10687(0)	.41115 03978 65469 30472(-2)
.97714 15146 39705 71416(0)	.46710 50372 11432 17474(-2)
.97218 28747 48581 79658(0)	.52491 23454 80885 91251(-2)
.96663 78515 58416 56709(0)	.58434 49875 83563 95076(-2)
.96049 12687 08020 28342(0)	.64519 00050 17573 69228(-2)
.95373 00064 25761 13641(0)	.70724 89995 43355 54680(-2)
.94634 28583 73402 90515(0)	.77033 75233 27974 18482(-2)
.93832 03977 79592 88365(0)	.83428 38753 96815 77056(-2)
.92965 48574 29740 05667(0)	.89892 75784 06413 57233(-2)
.92034 00254 70012 42073(0)	.96411 77729 70253 66953(-2)
.91037 11569 57004 29250(0)	.10297 11695 79563 55524(-1)
.89974 48997 76940 03664(0)	.10955 73338 78379 01648(-1)
.88845 92328 72256 99889(0)	.11615 72331 99551 34727(-1)
.87651 34144 84705 26974(0)	.12275 83056 00827 70087(-1)
.86390 79381 93690 47715(0)	.12934 83966 36073 73455(-1)
.85064 44947 68350 27976(0)	.13591 57100 97655 45790(-1)
.83672 59381 68868 73550(0)	.14244 87737 29167 74306(-1)
.82215 62543 64980 40737(0)	.14893 64166 48151 82035(-1)
.80694 05319 50217 61186(0)	.15536 77555 58439 82440(-1)
.79108 49337 99848 36143(0)	.16173 21872 95777 19942(-1)
.77459 66692 41483 37704(0)	.16801 93857 41038 65271(-1)
.75748 39663 80513 63793(0)	.17421 93015 94641 73747(-1)
.73975 60443 52694 75868(0)	.18032 21639 03912 86320(-1)
.72142 30853 70098 91548(0)	.18631 84825 61387 90186(-1)
.70249 62064 91527 07861(0)	.19219 90512 47277 66019(-1)
.68298 74310 91079 22809(0)	.19795 49504 80974 99488(-1)
.66290 96600 24780 59546(0)	.20357 75505 84721 59467(-1)
.64227 66425 09759 51377(0)	.20905 85144 58120 23852(-1)
.62110 29467 37226 40294(0)	.21438 98001 25038 67246(-1)
.59940 39302 42242 89297(0)	.21956 36630 53178 24939(-1)
.57719 57100 52045 81484(0)	.22457 26582 68160 98707(-1)
.55449 51326 31932 54887(0)	.22940 96422 93877 48761(-1)
.53131 97436 44375 62397(0)	.23406 77749 53140 06201(-1)
.50768 77575 33716 60215(0)	.23854 05210 60385 40080(-1)
.48361 80269 45841 02756(0)	.24282 16520 33365 99358(-1)
.45913 00119 89832 33287(0)	.24690 52474 44876 76909(-1)
.43424 37493 46802 55800(0)	.25078 56965 29497 68707(-1)
.40897 98212 29888 67241(0)	.25445 76996 54647 65813(-1)